Math 4
U4 L1 I3-I5 Review

① $\tan(x+3\pi) = \dfrac{\tan x + \tan(3\pi)}{1 - \tan x \cdot \tan(3\pi)}$

$= \dfrac{\tan x + 0}{1 - \tan x \cdot 0}$

$= \underline{\tan x}$

② $\cos\left(\frac{\pi}{2} - x\right) = \cos\frac{\pi}{2}\cdot\cos x + \sin\frac{\pi}{2}\cdot\sin x$

$= 0 \cdot \cos x + 1 \cdot \sin x$

$= \underline{\sin x}$

③ $\dfrac{\sin 2x}{\sin x} - \dfrac{\cos 2x}{\cos x} = \sec x$

$\dfrac{2\sin x\cdot\cos x}{\sin x} - \dfrac{2\cos^2 x - 1}{\cos x} =$

$\left(\dfrac{\cos x}{\cos x}\right)\dfrac{2\cos x}{1} - \dfrac{2\cos^2 x - 1}{\cos x} =$

$\dfrac{2\cos^2 x - 2\cos^2 x + 1}{\cos x} =$

$\dfrac{1}{\cos x} =$

$\underline{\sec x = \sec x}$

④ $\cos(195^\circ) = \cos(150 + 45)$

$= \cos 150\cos 45 + \sin 150\sin 45$

$= -\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} - \frac{1}{2}\cdot\frac{\sqrt{2}}{2}$

$= \underline{-\dfrac{\sqrt{6}-\sqrt{2}}{4}}$

⑤ $\tan(75^\circ)$

$\tan(45+30)$

$= \dfrac{\tan 45 + \tan 30}{1 - \tan 45\cdot\tan 30}$

$= \dfrac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}}$ or $\dfrac{\sqrt{3}+1}{\sqrt{3}-1}$

⑥ $\sin\left(\frac{17\pi}{12}\right) = \sin\left(\frac{9\pi}{12} + \frac{8\pi}{12}\right)$

$= \sin\left(\frac{3\pi}{4} + \frac{2\pi}{3}\right)$

$= \sin\frac{3\pi}{4}\cdot\cos\frac{2\pi}{3} + \cos\frac{3\pi}{4}\cdot\sin\frac{2\pi}{3}$

$= \frac{\sqrt{2}}{2}\cdot -\frac{1}{2} + -\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}$

$= \underline{-\dfrac{\sqrt{2}-\sqrt{6}}{4}}$

(7) $\sin 42 \cos 12 - \cos 42 \sin 12$

$= \sin(42-12)$

$= \sin 30$

$= \frac{1}{2}$

(8) $\cos(15)\cos 60 + \sin 15 \sin 60$

$\cos(15-60)$

$\cos(-45)$

$\frac{\sqrt{2}}{2}$

(9) $\sin\frac{\pi}{12} \cdot \cos\frac{5\pi}{12} + \cos\frac{\pi}{12} \cdot \sin\frac{5\pi}{12}$

$\sin\left(\frac{\pi}{12} + \frac{5\pi}{12}\right)$

$\sin\left(\frac{\pi}{12} + \frac{3\pi}{12}\right)$

$\sin\left(\frac{4\pi}{12}\right)$

$\sin\left(\frac{\pi}{3}\right)$

$\frac{\sqrt{3}}{2}$

(10) $\cos\frac{\pi}{16}\cos\frac{3\pi}{16} - \sin\frac{\pi}{16}\sin\frac{3\pi}{16}$

$\cos\left(\frac{\pi}{16} + \frac{3\pi}{16}\right)$

$\cos\left(\frac{4\pi}{16}\right)$

$\cos\left(\frac{\pi}{4}\right)$

$\frac{\sqrt{2}}{2}$

(11) $\dfrac{\tan 25 + \tan 110}{1 - \tan 25 \cdot \tan 110}$

$\tan(25 + 110)$

$\tan(135)$

-1

(12) $\dfrac{\tan\frac{5\pi}{4} - \tan\frac{\pi}{12}}{1 + \tan\frac{5\pi}{4} \cdot \tan\frac{\pi}{12}}$

$\tan\left(\frac{5\pi}{4} - \frac{\pi}{12}\right)$

$\tan\left(\frac{15\pi}{12} - \frac{\pi}{12}\right)$

$\tan\left(\frac{14\pi}{12}\right)$

$\tan\left(\frac{7\pi}{6}\right)$

$\frac{1}{\sqrt{3}}$

(13) $2\sin^2 x - \sin x - 1 = 0$

$(2\sin x + 1)(\sin x - 1) = 0$

$\sin x = -\frac{1}{2}$ | $\sin x = 1$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$ | $x = \frac{\pi}{2}$

(15) $2\cos x = -\sin 2x$

$2\cos x + \sin 2x = 0$

$2\cos x + 2\sin x \cdot \cos x = 0$

$2\cos x(1 + \sin x) = 0$

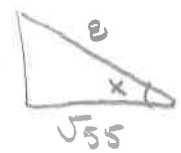
$2\cos x = 0$ | $1 + \sin x = 0$

$\cos x = 0$ | $\sin x = -1$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ | $x = \frac{3\pi}{2}$

(17) $\sin x = \frac{3}{8}$ $\quad x \in \text{QII}$

$\cos x = -\frac{\sqrt{55}}{8}$



$\cos 2x = \left(-\frac{\sqrt{55}}{8}\right)^2 - \left(\frac{3}{8}\right)^2$

$= \frac{55}{64} - \frac{9}{64}$

$= \frac{46}{64}$

$= \frac{23}{32}$

$\sin 2x = 2\left(\frac{3}{8}\right)\left(-\frac{\sqrt{55}}{8}\right)$

$= -\frac{6\sqrt{55}}{64}$

$= -\frac{3\sqrt{55}}{32}$

$\tan 2x = \frac{-\frac{3\sqrt{55}}{32}}{\frac{23}{32}}$

$= -\frac{3\sqrt{55}}{23}$

Quad IV $\begin{cases} \cos 2x \text{ pos} \\ \sin 2x \text{ neg} \end{cases}$ $\Rightarrow$

(18) $\frac{\tan x}{1 + \tan^2 x} = \sin x \cdot \cos x$

$\frac{\frac{\sin x}{\cos x}}{\sec^2 x} =$

$\frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos^2 x}} =$

$\frac{\sin x}{\cos x} \cdot \frac{\cos^2 x}{1} =$

$\sin x \cdot \cos x = \sin x \cdot \cos x$

(14) $\sin x + \sin x \cdot \cot x = 0$

$\sin x(1 + \cot x) = 0$

$\sin x = 0$ | $1 + \cot x = 0$

$x = 0, \pi, 2\pi$ | $\cot x = -1$

$\tan x = -1$

$x = \frac{3\pi}{4}, \frac{7\pi}{4}$

(16) $-2\sin^2 x - 5\cos x + 4 = 0$

$-2(1 - \cos^2 x) - 5\cos x + 4 = 0$

$-2 + 2\cos^2 x - 5\cos x + 4 = 0$

$2\cos^2 x - 5\cos x + 2 = 0$

$(2\cos x - 1)(\cos x - 2) = 0$

$\cos x = \frac{1}{2}$ $\quad$ $\cos x = 2$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$ $\quad$ No solution

(19) $\tan^2 + 1 + \tan x \cdot \sec x = \frac{1 + \sin x}{\cos^2 x}$

$\sec^2 x + \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} =$

$\frac{1}{\cos^2 x} + \frac{\sin x}{\cos^2 x} =$

$\frac{1 + \sin x}{\cos^2 x} = \frac{1 + \sin x}{\cos^2 x}$